The Gateway Primary Academy
Calculation Policy

Maths Mastery

At the centre of the mastery approach to the teaching of maths is the belief that all children have the potential to succeed. They should have access to the same curriculum content and, rather than being extended with new learning, they should deepen their conceptual understanding by tackling challenging and varied problems. Similarly, with calculation strategies, children must not simply rote learn procedures but demonstrate their understanding of these procedures through the use of concrete materials and pictorial representations. This policy outlines the different calculation strategies that should be taught and used across the school, which is in line with the requirements of the 2014 Primary National Curriculum.

Mathematical Language

The 2014 National Curriculum is explicit in articulating the importance of children using the correct mathematical language as a central part of their learning (reasoning). In certain year groups, the non-statutory guidance highlights the requirement for children to extend their language around certain concepts. It is therefore essential that teaching using the strategies outlined in this policy is accompanied by the use of appropriate and precise mathematical vocabulary. New vocabulary should be introduced in a suitable context (for example, with relevant, real objects, apparatus, pictures of diagrams) and explained carefully. High expectations of the mathematical language used are essential, with teachers only accepting what is correct

> 'The quality and variety of language that pupils hear and speak are key factors in developing their mathematical vocabulary and presenting a mathematically justification, argument or proof.'
>
> – 2014 Mathematics Programme of Study

This policy has been designed to teach children through the use of concrete, pictorial and abstract methods. This calculation policy should be used to support children to develop a deep understanding of number and calculation.

Using the Concrete-Pictorial-Abstract Approach:

Children develop an understanding of a mathematical concept through the three steps of: concrete, pictorial and abstract approach. Reinforcement is achieved by going back and forth between these representations.

Concrete Representation:

This is the first step in a child's learning. The child is introduced to an idea or skill by acting it out with real objects. This is a 'hands on' component using real objects and it is the foundation for conceptual understanding.

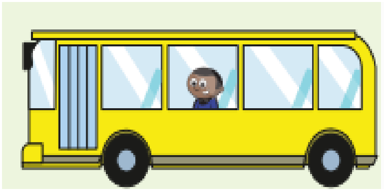
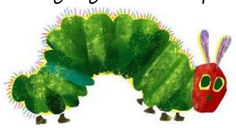
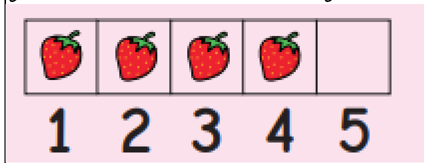
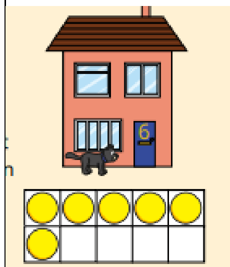
Pictorial Representation:

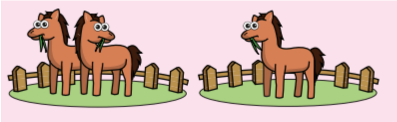
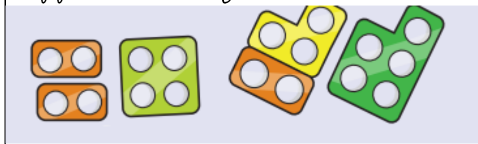
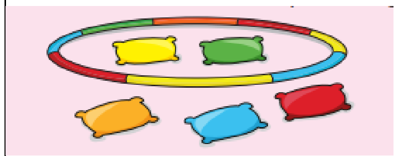
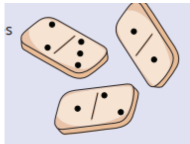
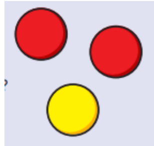
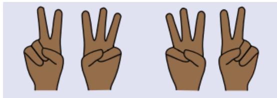
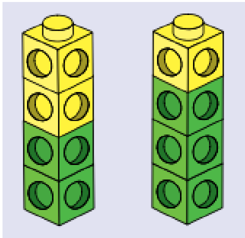
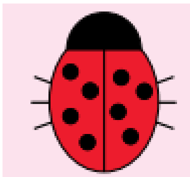
Once the child has sufficiently understood the 'hands on' experience, they can be progressed onto relating them to pictorial representations, such as a diagram or a picture of the problem.

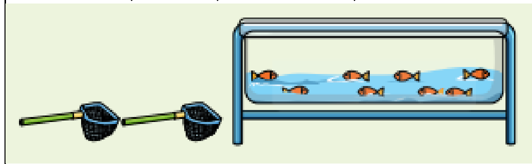
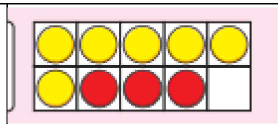
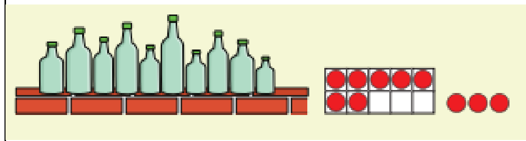
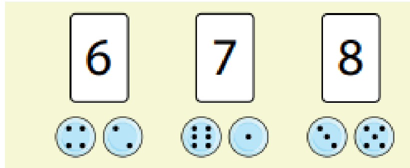
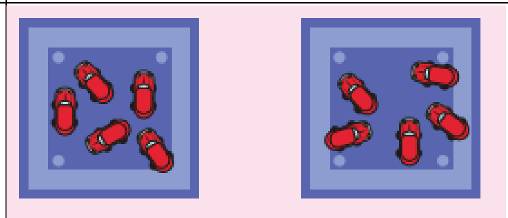
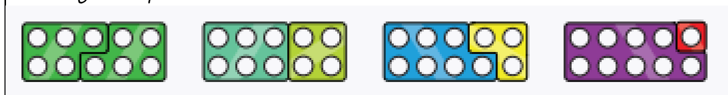
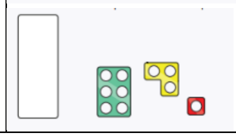
Abstract Representation:

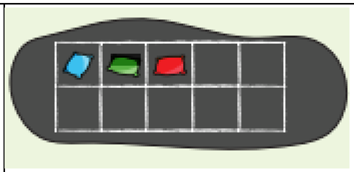
This is the third step in a child's learning. The child should now be capable of representing problems by using mathematical notation, for example: $12 \div 2 = 6$

Addition - Year R

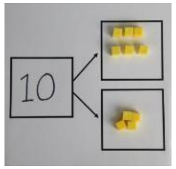
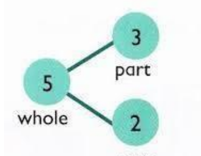
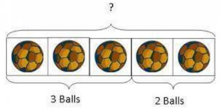
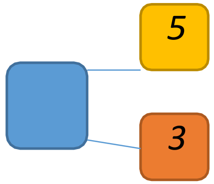
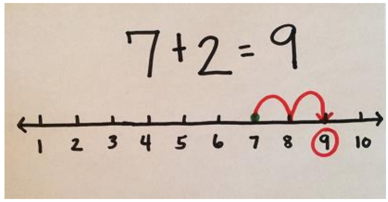
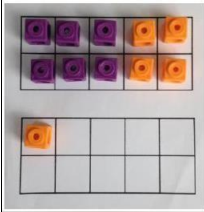
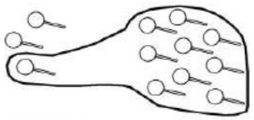
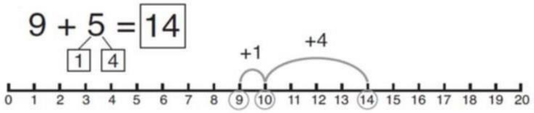
Objective & Strategy	Concrete	Pictorial	Abstract
Recognise one more Once children are confident with one more, extend to two more	Children respond to rhymes and use practical resources to act them out e.g. The Gingerbread man - how many characters have you got now?  Using cubes and mathematical equipment to add one more to a tower.  Play games involving 1 more e.g. what's the time Mr Wolf, add 1 more step on..  Engage in roleplay activities linked to one more e.g. a bus route around the outside area - how many are on the bus now? 	Explore stories alongside visual representations e.g. The Very Hungry Caterpillar  Begin to recognise 1 more on a 5 frame and recognise how many you will have altogether.  Begin to use tens frames to represent stories e.g. Six Dinner's Sid 	Begin to recognise the addition sign and begin to recognise simple addition number sentences e.g. 4 + 1 =

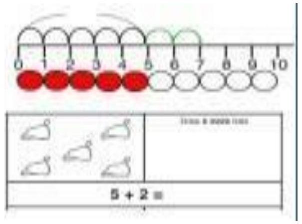
Recognise the composition of numbers	 In small world play, children recognise how the horses make 3. Use practical resources to make numbers to 10 in different ways.  Explore practically partitioning numbers in different ways.  Use numicon to explore adding 2 numbers together - how many do you get as the total?  Engage in games outside - how many are in the hoop - how many are outside?	 Children recognise how the dominoes represent numbers in different ways.  Exploring making amounts using double sided counters.  Use fingers to represent 5 in different ways  Explore how you can make 5 using different colours visually.  How is 8 represented visually? How many ways can you represent 8?	Begin to recognise the addition sign and begin to recognise simple addition number sentences e.g. 4 + 1 =

Combine 2 groups	 Explore combining two groups using visual prompts and practical resources  Play games in continuous provision such as: fishing. How many fish does each person have? How many do they have altogether? Read and discuss stories involving addition e.g. Mouse Count. Engage in role play linked to mathematical stories e.g. Mr Grumpy's Day Out.	 Represent an addition question using a tens frame (6 + 3 = 9)  Begin to recognise the link between number rhymes and more abstract representations.	 Start to choose 2 number cards to add together. Make the number visually and find the total.
Begin to develop an understanding of number bonds to 10	 Children explore number bonds to 10 using practical resources e.g. how many ways can you park these 10 small world cars?   Use mathemtical equipment to explore making numbers to 10	Children draw around the numicon shapes within a template of a ten numicon piece to see which ones will fit inside to make 10, then colour them in.	Children begin to fill in number sentences with the numbers missing to get used to the format.

	 Use a large tens frame on the playground to explore collecting objects and making 10 using twigs and leaves for example.		
Compare numbers to 10 to develop an understanding of the terms 'most' and 'greater'			

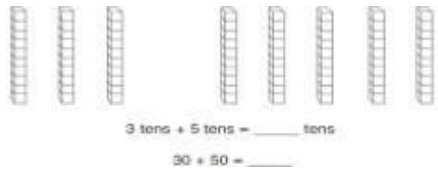
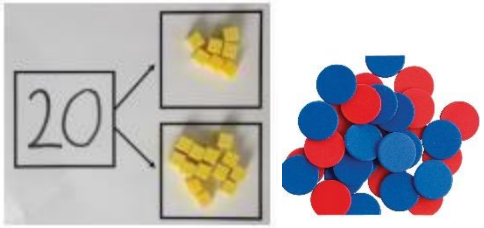
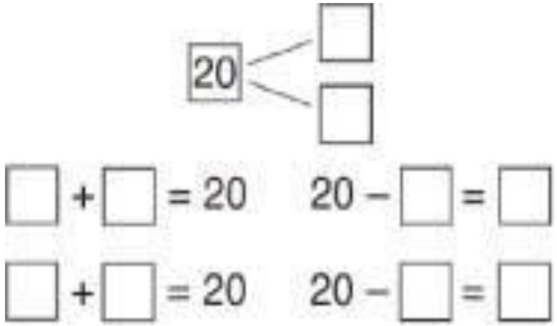
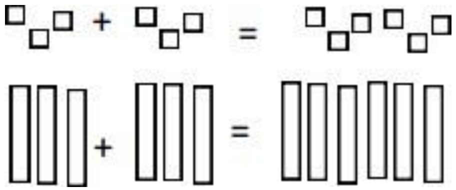
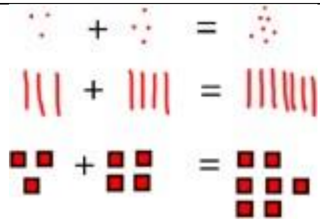
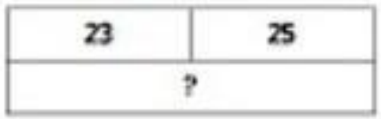
Addition - Year 1

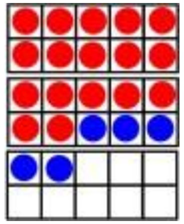
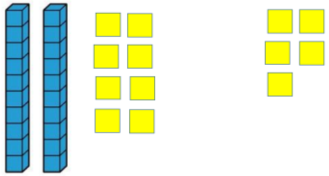
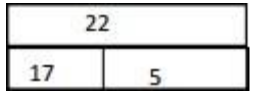
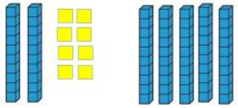
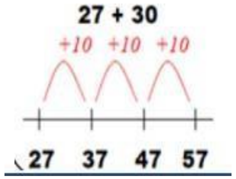
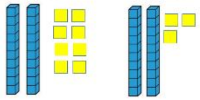
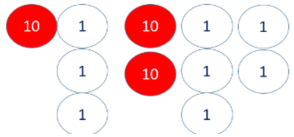
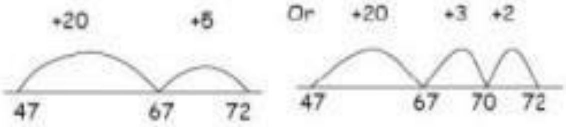
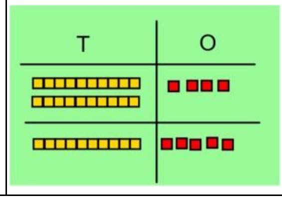
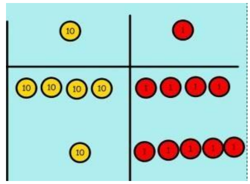
Objective & Strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model.	   Use part- part whole model. Use cubes to add two numbers together as a group or in a bar. 	    Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 10 = 6 + 4  Use the part-part whole diagram as shown above to move into the abstract.
Starting at the bigger number and counting on.	 Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	7 + 2= 9  Start at the larger number on the number line and count on in ones.	12 + 5 = 17 Place the larger number in your head and count on the smaller number to find your answer.
Regrouping to make 10.	6 + 5 = 11  	 3 + 9 = Use pictures or a number line. Regroup or partition the smaller number to make 10. 	7 + 4 = 11 "If I am at seven, how many more do I need to make 10? How many more do I add on now?"

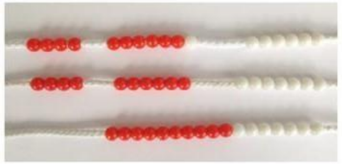
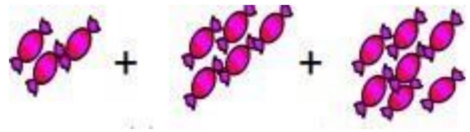
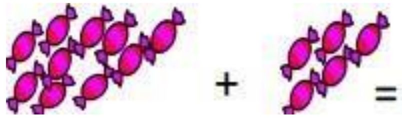
	Start with the bigger number and use the smaller number to make 10.		
Represent & use number bonds and related subtraction facts within 20.	 2 more than 5.	 	Emphasis should be on the language: "1 more than 5 is equal to 6" "2 more than 5 is 7" "8 is 3 more than 5"
Mental strategies taught through Number Sense over the year			

One More, One Less	When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. 5 – 1 = 4).	Number Neighbours: Spot the Difference	Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2. Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. 5 – 4 = 1), of adjacent odds (e.g. 9 – 7 = 2) or adjacent evens (e.g. 6 – 4 = 2)
Two More, Two Less: Think Odds and Evens	If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.	7 Tree and 9 Square	Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that 7 – 3 = 4. Visualising the 9 square helps remember that 3 + 6 = 9.
Number 10 Fact Families	Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.	Ten and A Bit	The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. 3 + 10 = 13, 17 – 7 = 10, 12 – 10 = 2).
Five and A Bit	The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. 5 + 3 = 8, 9 – 5 = 4).	Make Ten and Then…	Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. 8 + 6 can be calculated by thinking '8 + 2 = 10 and 4 more makes 14'). The same strategy can be applied to subtractions through 10.
Know about 0 0	When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.	Adjust It	Any addition and subtraction can be calculated by adjusting from a fact you know already, (e.g. 6 + 9 is one less than 6 + 10).
Doubles and Near Doubles	Memorise doubles of numbers to 10, using a visual approach. Then use these known double facts to calculate near doubles and hidden doubles. Once we know 6 + 6 = 12 then 6 +7 and 5 + 7 is easy.	Swap It 1 + 6	When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. 1 + 8 = 8 + 1. Sometimes reversing the order of the two addends makes addition easier to think about conceptually.

Addition Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Adding multiples of ten.	50 = 30 + 20  Model using dienes and bead strings.	 Use representations for base ten.	20 + 30 = 50 70 = 50 + 20 40 + _ = 60
Use known number facts including different combinations of tens & ones of any 2 digit number. (Part part whole)	 Children explore ways of making numbers.	 	Include teaching of the inverse of addition and subtraction: ☐ + 1 = 16 16 – 1 = ☐ 1 + ☐ = 16 16 – ☐ = 1
Use known facts.		 Children draw representations of H, T & O.	3 + 4 = 7 Leads to 30 + 40 = 70 Leads to 300 + 400 = 700
Use bar models.	 3 + 4 = 7	 7 + 3 = 10	 23 + 25 = 48

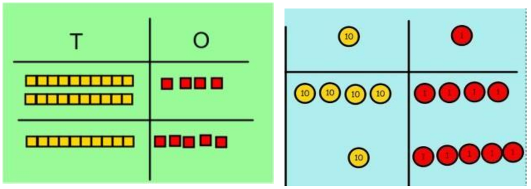
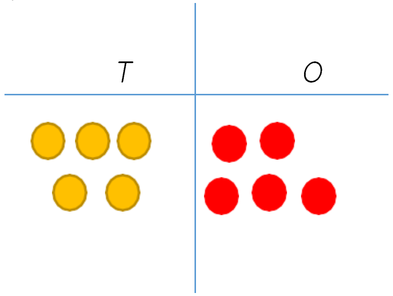
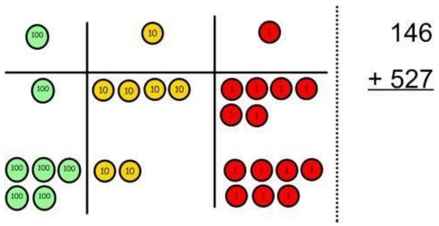
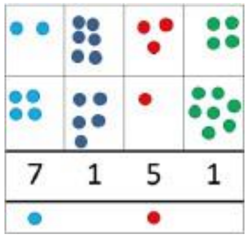
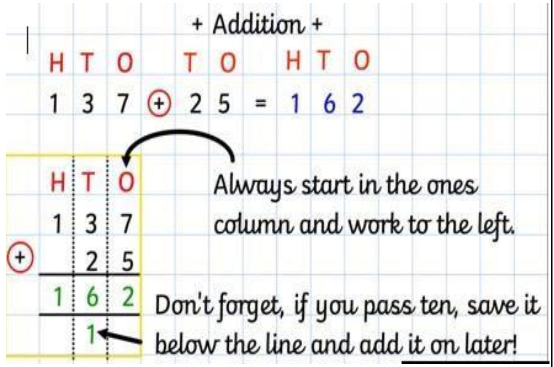
Add a two digit number and ones.	 17 + 5 = 22 Use ten frame to make 'magic ten'. Children explore the patterns: 17 + 5 = 22 27 + 5 = 32	2 8 + 5 = 33  Draw out pictorial representation of diennes or place value counters	17 + 5 = 22 Explore related facts: 17 + 5 = 22 5 + 17 = 22 22 - 17 = 5 22 - 5 = 17 
Add 2 digit numbers and tens.	 25 + 10 = 35 Explore that the ones digit does not change.	2 8 + 5 0 = 78   Draw out a pictorial representation initially and then move onto using blank number lines when secure in preparation for Year 3	27 + 10 = 37 27 + 20 = 47 27 + ___ = 57
Add two 2-digit numbers.	 Model using dienes, place value counters and numicon.	2 8 + 2 3 = 51  13 + 25 = 28  Draw out pictorial representations initially and then move onto using a blank number line and bridge ten using part whole if necessary.  	25 + 47 20 + 5 40 + 7 20 + 40 = 60 5+ 7 =12 60 + 12 = 72 Children begin to be introduced to the column method:  

Add three 1-digit numbers.	4 + 7 + 6 = 17 Put 4 and 6 together to make 10. Add on 7.  Following on from making 10, make 10 with 2 of the digits (if possible) then add on the third digit.	 Regroup and draw representation.  = 15	$4 + 7 + 6 = \boxed{10} + \boxed{7}$ 10 $= \boxed{17}$ Combine the two numbers that make/bridge ten, then add on the third.

Mental strategies taught through Number Sense over the year

One More, One Less	When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. 5 – 1 = 4).	Number Neighbours: Spot the Difference	Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2. Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. 5 – 4 = 1), of adjacent odds (e.g. 9 – 7 = 2) or adjacent evens (e.g. 6 – 4 = 2)
Two More, Two Less: Think Odds and Evens	If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.	7 Tree and 9 Square	Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that 7 – 3 = 4. Visualising the 9 square helps remember that 3 + 6 = 9.
Number 10 Fact Families	Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.	Ten and A Bit	The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. 3 + 10 = 13, 17 – 7 = 10, 12 – 10 = 2).
Five and A Bit	The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. 5 + 3 = 8, 9 – 5 = 4).	Make Ten and Then...	Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. 8 + 6 can be calculated by thinking '8 + 2 = 10 and 4 more makes 14'). The same strategy can be applied to subtractions through 10.
Know about 0	When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.	Adjust It	Any addition and subtraction can be calculated by adjusting from a fact you know already, (e.g. 6 + 9 is one less than 6 + 10).
Doubles and Near Doubles	Memorise doubles of numbers to 10, using a visual approach. Then use these known double facts to calculate near doubles and hidden doubles. Once we know 6 + 6 = 12 then 6 +7 and 5 + 7 is easy.	Swap It	When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. 1 + 8 = 8 + 1. Sometimes reversing the order of the two addends makes addition easier to think about conceptually.

Addition Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Column Addition – no regrouping Add 2 or 3 digit numbers.	24 + 15 =  Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. 	Add the ones first, then the tens, then the hundreds: $$\begin{array}{r} 223 \\ +\ 114 \\ \hline 337 \end{array}$$
Column Addition - with regrouping.	Make both numbers on a place value grid.  Add up the units and exchange 10 ones for one 10.	Children can draw a pictorial representation of the columns and place value counters to further support their learning and understanding. 	Children regroup and recognise they need to add the ones first and then tens/hundreds. 

<table>
<tr>
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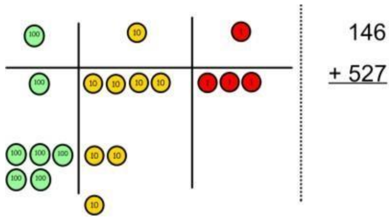


Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added.

This can also be done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100.

As children move on to decimals, money and decimal place value counters can be used to support learning.

</td>
<td></td>
<td>

As the children move on, introduce decimals with the same number of decimal places and different. Money is used for context.

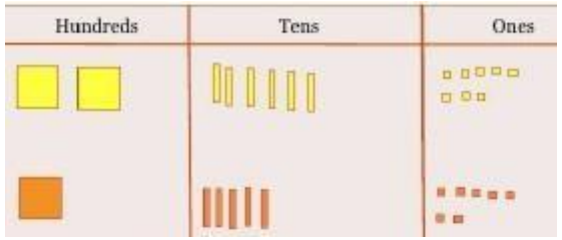
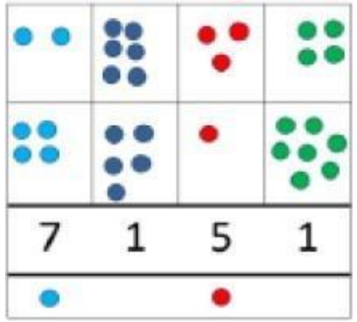
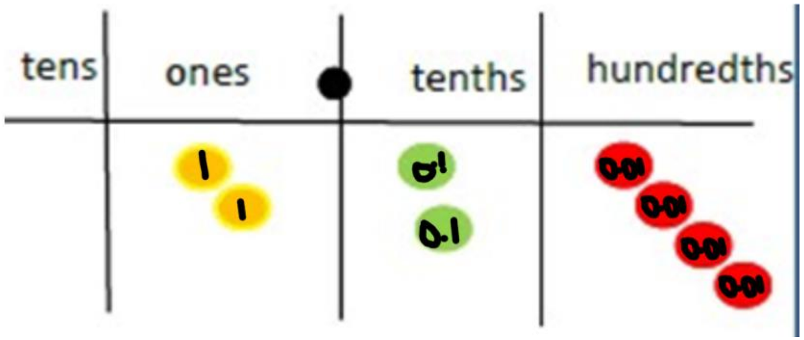
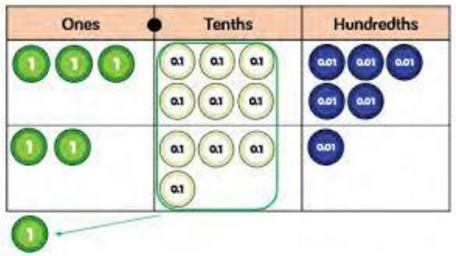
$\begin{array}{r} 72.8 \\ +\ 54.6 \\ \hline 127.4 \\ \hline 1\ 1\ \ \ \ \end{array}$

$\begin{array}{rcccccc} & £ & 2 & 3 & . & 5 & 9 \\ + & £ & & 7 & . & 5 & 5 \\ \hline & £ & 3 & 1 & . & 1 & 4 \\ \hline & & 1 & 1 & & 1 & \end{array}$

$\begin{array}{cccccc} 2 & 3 & . & 3 & 6 & 1 \\ & 9 & . & 0 & 8 & 0 \\ 5 & 9 & . & 7 & 7 & 0 \\ + & 1 & . & 3 & 0 & 0 \\ \hline 9 & 3 & . & 5 & 1 & 1 \\ \hline 2 & 1 & & 2 & & \end{array}$

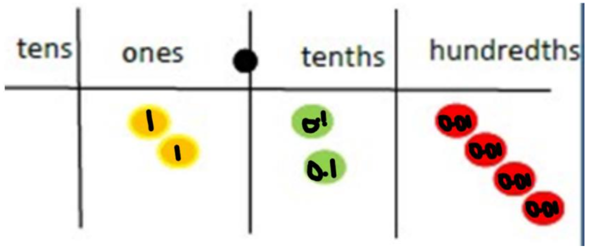
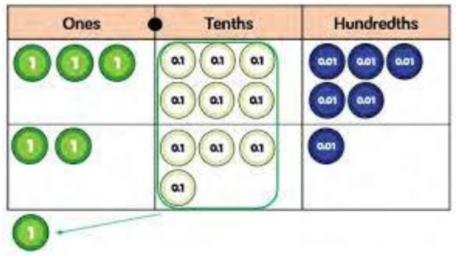
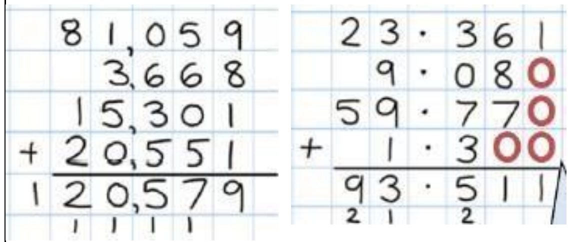
</td>
</tr>
</table>

Addition Years 4 - 6

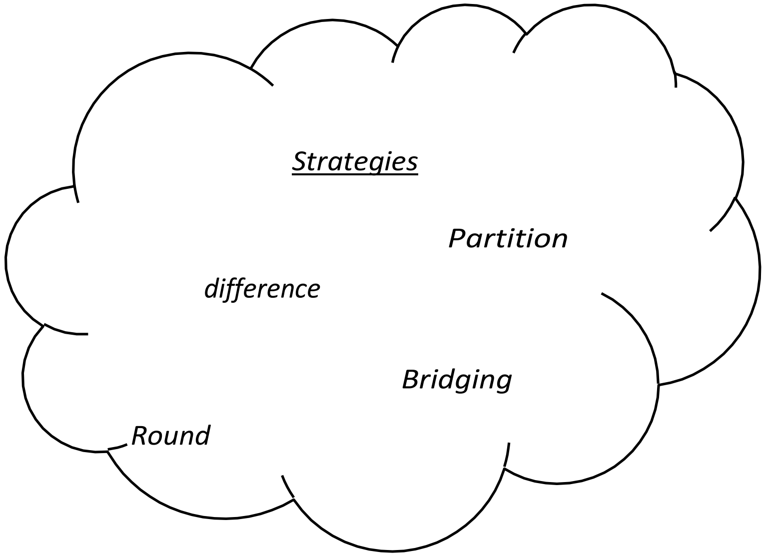
Objective & Strategy	Concrete	Pictorial	Abstract
<u>Year 4</u> Add numbers with up to 4 digits	Children continue to use dienes or place value counters to add, exchanging ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand. 	Draw representations using place value grid. 	Continue from previous work to carry hundreds as well as tens. Relate to money and measures. Th H T I 3 5 1 7 + 0 3 9 6 3 9 1 3 1 1
<u>Year 5</u> Add numbers with more than 4 digits. Add decimals with 2 decimal places, including money.	(As year 4) Introduce decimal place value counters and model exchange for addition. Match to place value counters 	(As year 4) 	(As year 4) 72.8 + 54.6 127.4 1 1 £ 2 3 . 5 9 + £ 7 . 5 5 £ 3 1 . 1 4 1 1 1

<table>
<tr>
<td><u>Year 6</u>
Add several numbers of increasing complexity.

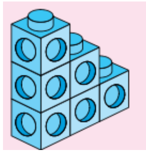
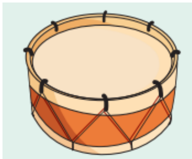
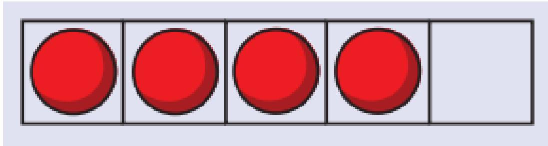
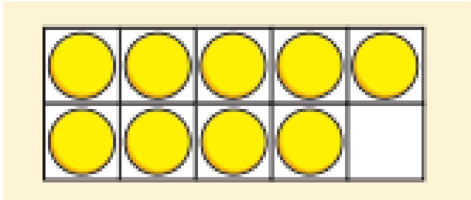
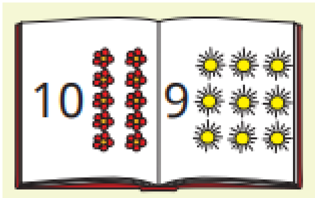
Include adding money, measure and decimals with different numbers of decimal points.</td>
<td>(As year 5)

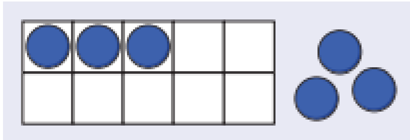
Introduce decimal place value counters and model exchange for addition.
</td>
<td>(As year 5)
</td>
<td></td>
</tr>
<tr>
<td><u>Year 4</u> Rapid Recall (addition and subtraction)</td>
<td colspan="3">

- Sums/differences – multiples of 10/100/1000
- Doubles – within 100
- Add/subtract multiples of 10/100/1000

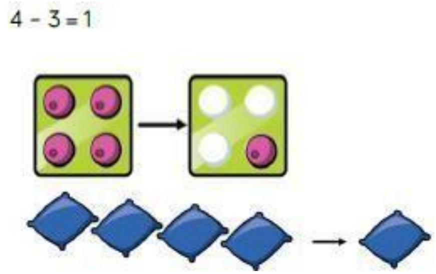
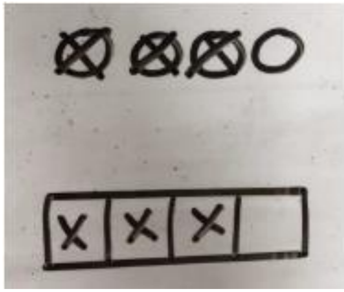
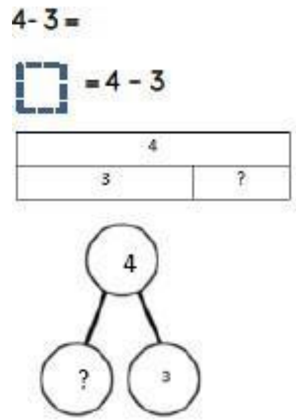
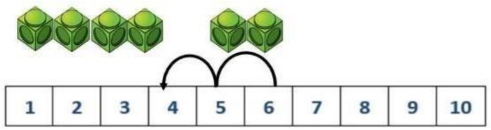
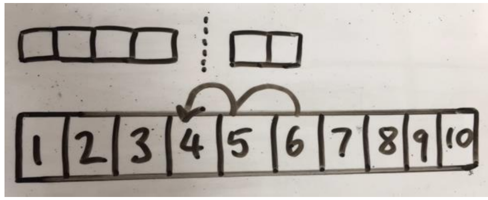
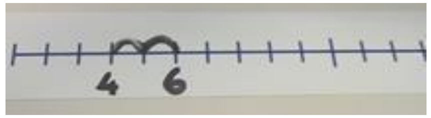
</td>
</tr>
</table>

Subtraction - Year R

Objective & Strategy	Concrete	Pictorial	Abstract
Recognise one less Once children are confident with one less, extend to two less	 Children respond to rhymes and use practical resources to act them out e.g. Five little speckled frogs, five current buns  Using cubes and mathematical equipment to notice a pattern and discuss what happens when you take one away.  Play games involving 1 less e.g. hitting a drum. How many is 1 less? Engage in role play, such as a role play shop to pay for items	Begin to link concrete items e.g. 5 current buns to pictorial representations. How many currant buns are left?  Begin to use a tens frame to recognise 1 less linked to a story:  Children represent 1 less visually and create their own book: 	Begin to ask the children to take one away in their heads e.g. drop 5 pebbles into a container - how many would be left if I take one away? Begin to count backwards following reading a text e.g. Stella to Earth! Begin to recognise the takeaway symbol when working with an adult modelling number sentence writing.
Understand the concept of zero	 Explore through a range of nursery rhymes the idea of everything being gone.	Go on a zero hunt around school. Use the term zero when playing and exploring in their environment.	Represent 0 using a number.

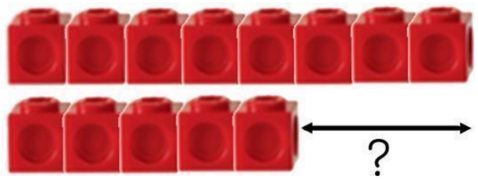
	 Read a range of stories which discuss the concept of zero e.g. Zero is the leaves on the Tree		
Recognise how to 'take away'	Use small world to explore taking away linked to a story Engage in story telling and number rhymes to act out taking away e.g. 10 currant buns.	 Use a tens frame to represent taking away Crossing out pictures on a frame to show them being taken away.	Begin to recognise the takeaway symbol with an adult, showing the difference between addition and subtraction.

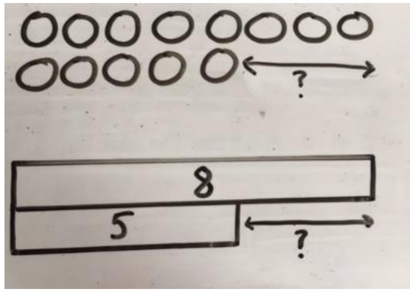
Year 1 Subtraction

Objective & Strategy	Concrete	Pictorial	Abstract
Taking away ones from a whole.	Use physical objects, counters, cubes etc. to show how objects can be taken away. 	Cross out drawn objects to show how many has been taken away. The bar model can also be used. 	
Counting back.	Counting back (using number lines or number tracks) children start with 6 and count back 2. 6 - 2 = 4 	Children to represent what they see pictorially e.g.  Pupils draw on the jumps but not the actual number line.	Children to represent the calculation on a number line or number track and show their jumps. Encourage children to use an empty number line.  

<table>
<tr>
<td>Finding the difference.</td>
<td>Finding the difference (using cubes, Numicon or Cuisenaire rods, other objects can also be used).

Calculate the difference between 8 and 5:

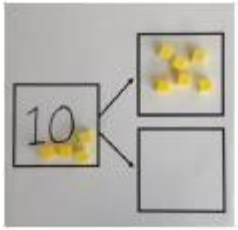

</td>
<td>Children to draw the cubes/other concrete objects which they have used or use the bar model to illustrate what they need to calculate.


</td>
<td>Find the difference between 8 and

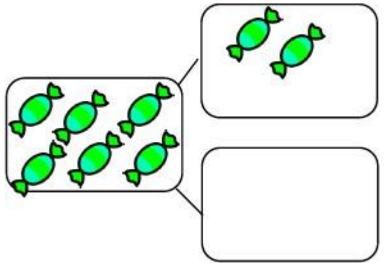
5. 8 - 5, the differences is…

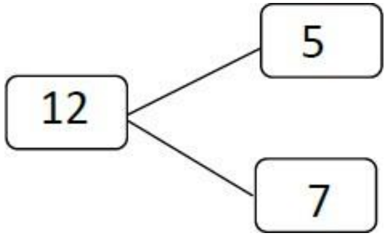
Children to explore why
9 - 6 = 8 - 5 = 7 - 4 have the same difference.</td>
</tr>
<tr>
<td>Represent and use number bonds and related subtraction facts within 20.

(Part part whole model)</td>
<td>Link to addition - use the PPW model to model the inverse.



If 10 is the whole and 6 is one of the parts, what is the other part?
10 - 6 = 4</td>
<td>Use pictorial representations to show the parts.

</td>
<td>Move to using numbers within the part whole model.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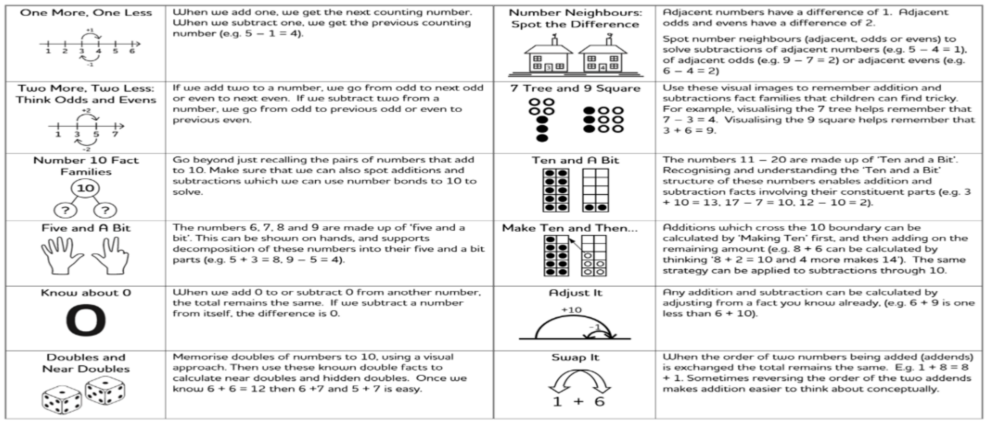
<tr>
<td>Mental strategies taught through Number Sense over the year</td>
<td colspan="3">
<table>
<tr><td>One More, One Less</td><td>When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. 5 – 1 = 4).</td><td>Number Neighbours: Spot the Difference</td><td>Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2.
Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. 5 – 4 = 1), of adjacent odds (e.g. 9 – 7 = 2) or adjacent evens (e.g. 6 – 4 = 2)</td></tr>
<tr><td>Two More, Two Less: Think Odds and Evens</td><td>If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.</td><td>7 Tree and 9 Square</td><td>Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that 7 – 3 = 4. Visualising the 9 square helps remember that 3 + 6 = 9.</td></tr>
<tr><td>Number 10 Fact Families</td><td>Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.</td><td>Ten and A Bit</td><td>The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. 3 + 10 = 13, 17 – 7 = 10, 12 – 10 = 2).</td></tr>
<tr><td>Five and A Bit</td><td>The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. 5 + 3 = 8, 9 – 5 = 4).</td><td>Make Ten and Then...</td><td>Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. 8 + 6 can be calculated by thinking '8 + 2 = 10 and 4 more makes 14'). The same strategy can be applied to subtractions through 10.</td></tr>
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O</td><td>When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.</td><td>Adjust It</td><td>Any addition and subtraction can be calculated by adjusting from a fact you know already, (e.g. 6 + 9 is one less than 6 + 10).</td></tr>
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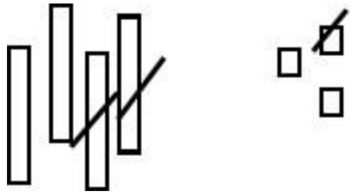
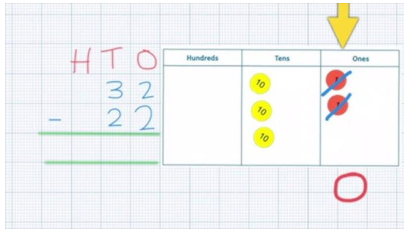
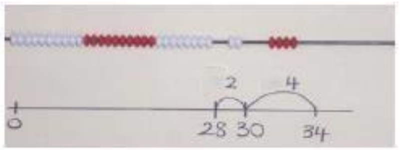
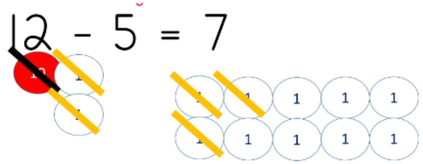
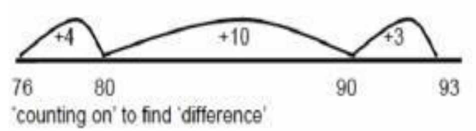
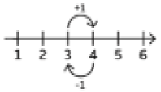
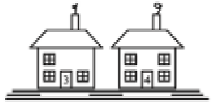
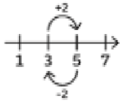
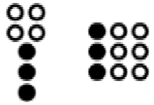
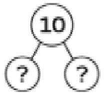
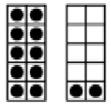
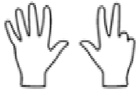
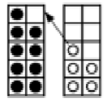
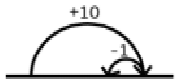
1 + 6</td><td>When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. 1 + 8 = 8 + 1. Sometimes reversing the order of the two addends makes addition easier to think about conceptually.</td></tr>
</table></td>
</tr>
</table>

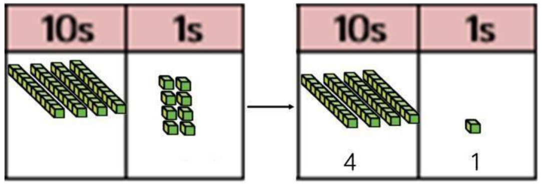
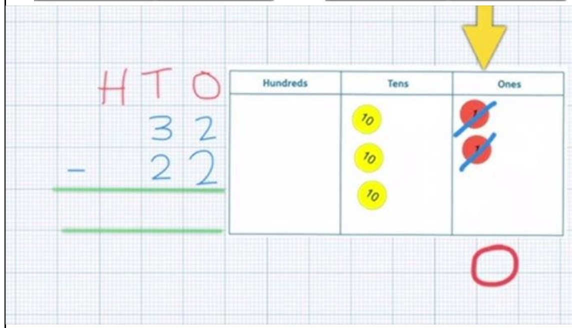
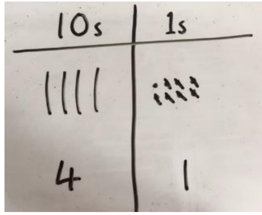
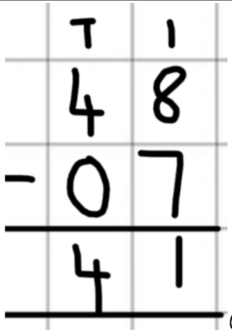
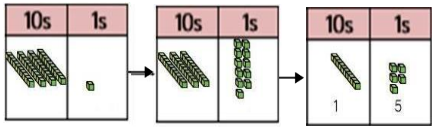
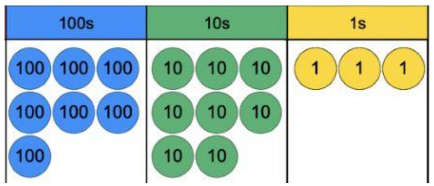
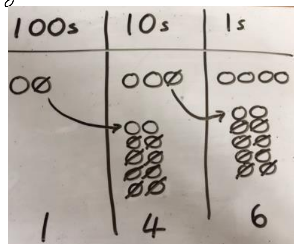
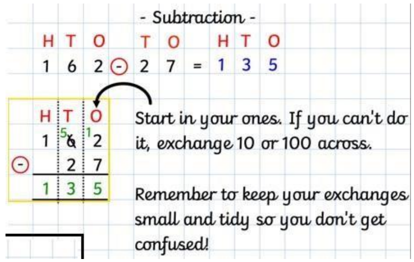
Subtraction Year 2

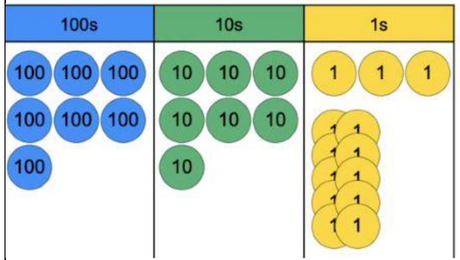
Objective & Strategy	Concrete	Pictorial	Abstract
Partitioning to subtract – without regrouping. (friendly numbers)	Use dienes to show how to partition the number when subtracting without regrouping. 34 - 13 = 21 	Children draw representations of dienes and cross off. 43 - 21 = 22 	 43 - 21 = 22
Making ten. (crossing one ten, crossing more than one ten, crossing the hundreds)	Use a bead string to model counting to the next ten and the rest. 34 - 28 = 	 Use pictorial representations to represent exchanging initially. Move onto using a number line to count on to the next ten and then the rest. 	93 - 76 = 17

Mental strategies taught through Number Sense over the year

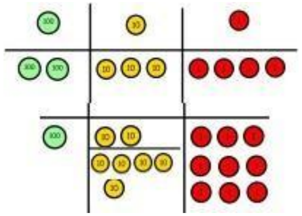
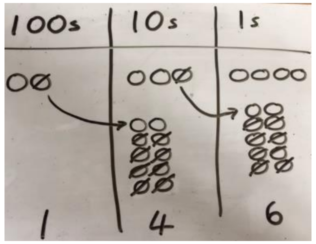
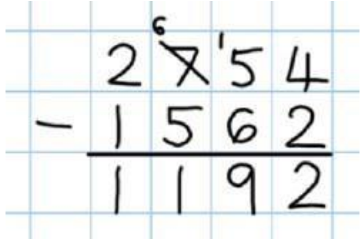
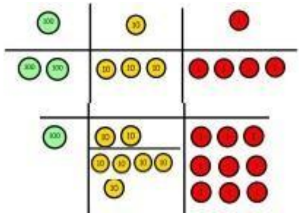
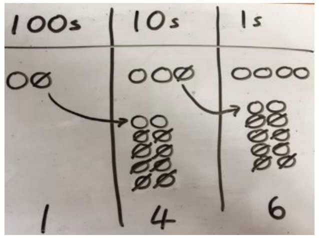
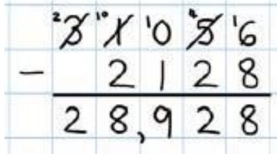
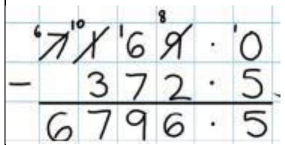
One More, One Less 	When we add one, we get the next counting number. When we subtract one, we get the previous counting number (e.g. 5 – 1 = 4).	Number Neighbours: Spot the Difference 	Adjacent numbers have a difference of 1. Adjacent odds and evens have a difference of 2. Spot number neighbours (adjacent, odds or evens) to solve subtractions of adjacent numbers (e.g. 5 – 4 = 1), of adjacent odds (e.g. 9 – 7 = 2) or adjacent evens (e.g. 6 – 4 = 2)
Two More, Two Less: Think Odds and Evens 	If we add two to a number, we go from odd to next odd or even to next even. If we subtract two from a number, we go from odd to previous odd or even to previous even.	7 Tree and 9 Square 	Use these visual images to remember addition and subtractions fact families that children can find tricky. For example, visualising the 7 tree helps remember that 7 – 3 = 4. Visualising the 9 square helps remember that 3 + 6 = 9.
Number 10 Fact Families 	Go beyond just recalling the pairs of numbers that add to 10. Make sure that we can also spot additions and subtractions which we can use number bonds to 10 to solve.	Ten and A Bit 	The numbers 11 – 20 are made up of 'Ten and a Bit'. Recognising and understanding the 'Ten and a Bit' structure of these numbers enables addition and subtraction facts involving their constituent parts (e.g. 3 + 10 = 13, 17 – 7 = 10, 12 – 10 = 2).
Five and A Bit 	The numbers 6, 7, 8 and 9 are made up of 'five and a bit'. This can be shown on hands, and supports decomposition of these numbers into their five and a bit parts (e.g. 5 + 3 = 8, 9 – 5 = 4).	Make Ten and Then... 	Additions which cross the 10 boundary can be calculated by 'Making Ten' first, and then adding on the remaining amount (e.g. 8 + 6 can be calculated by thinking '8 + 2 = 10 and 4 more makes 14'). The same strategy can be applied to subtractions through 10.
Know about 0 	When we add 0 to or subtract 0 from another number, the total remains the same. If we subtract a number from itself, the difference is 0.	Adjust It 	Any addition and subtraction can be calculated by adjusting from a fact you know already, (e.g. 6 + 9 is one less than 6 + 10).
Doubles and Near Doubles 	Memorise doubles of numbers to 10, using a visual approach. Then use these known double facts to calculate near doubles and hidden doubles. Once we know 6 + 6 = 12 then 6 +7 and 5 + 7 is easy.	Swap It  1 + 6	When the order of two numbers being added (addends) is exchanged the total remains the same. E.g. 1 + 8 = 8 + 1. Sometimes reversing the order of the two addends makes addition easier to think about conceptually.

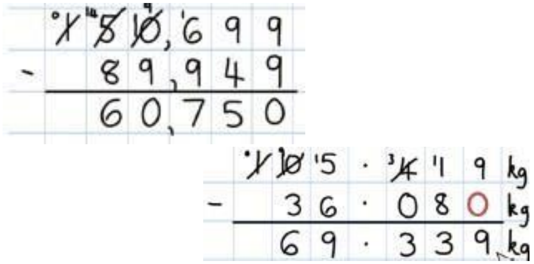
Subtraction Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping.	Column method using base ten.  	Children to represent the base 10 pictorially. 	 Column method or children could count back 7.
Column subtraction with regrouping.	Column method using base 10 and having to exchange. 41 - 26 =  783 - 458 = 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column method. Children must understand what has happened when they have crossed out digits. 

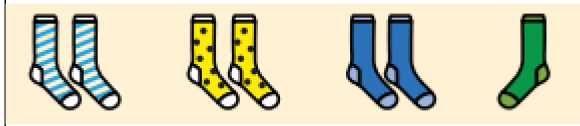
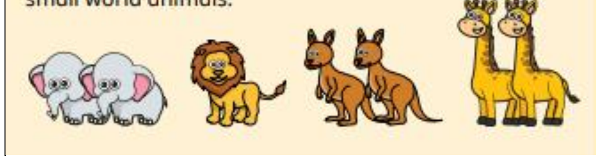
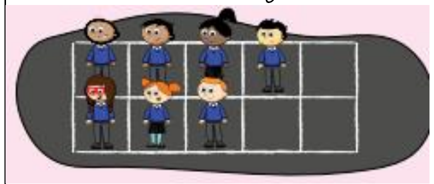
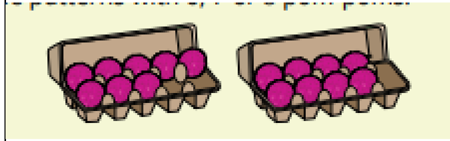
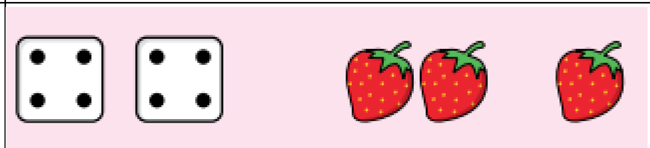
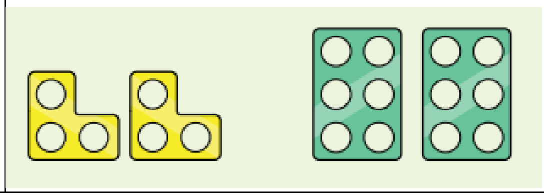
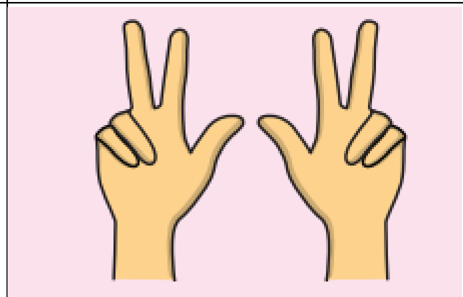
	100s 10s 1s		

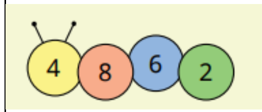
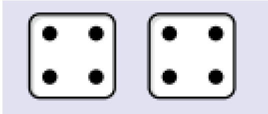
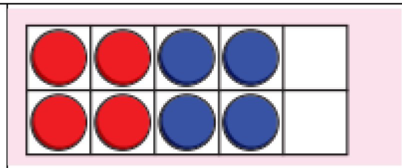
Subtraction Year 4 - 6

Objective & Strategy	Concrete	Pictorial	Abstract
<u>Year 4</u> Subtracting tens and ones - up to 4 digits. (introduce decimal subtraction through context of money)	Model process of exchange using place value counters. 234 - 179 = 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column method. Children must understand what has happened when they have crossed out digits. 
<u>Year 5</u> Subtract with at least 4 digits, including money and measures. (subtract with decimal values, including mixtures of integers and decimals and aligning the decimal)	Model process of exchange using place value counters. 234 - 179 = 	Represent the place value counters pictorially; remembering to show what has been exchanged. 	Formal column method. Children must understand what has happened when they have crossed out digits. Use zeros for place holders. Numbers over 10, 000 to have commas. 

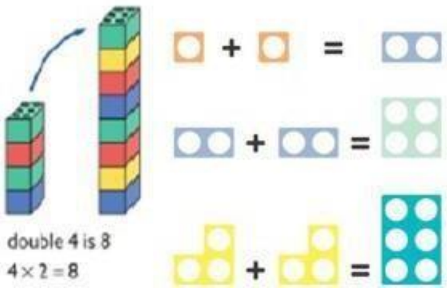
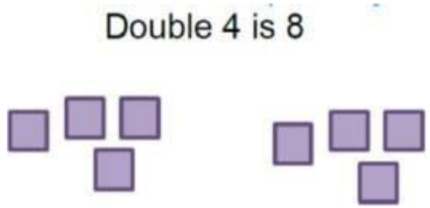
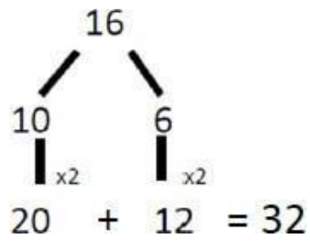
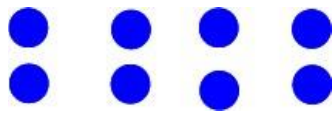
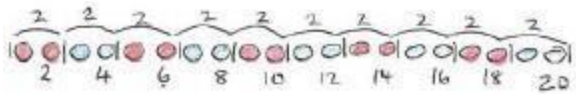
<u>Year 6</u> Subtract with increasingly large, more complex, numbers and decimal values.			Increasingly large and more complex numbers. 

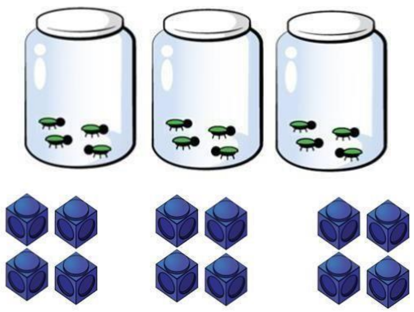
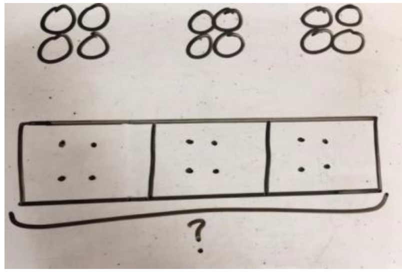
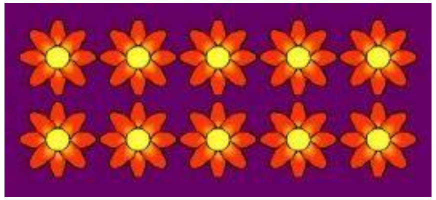
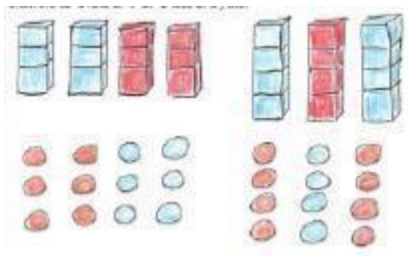
Multiplication - Year R

Objective & Strategy	Concrete	Pictorial	Abstract
Begin to understand the concept of a pair	 Read stories e.g. Simon Sock and model making pairs with objects. Children understand the concept of a pair being 2.  Act out traditional stories such as: Noah's ark using small world  Take part in large group activities such as: filling a large tens frame on the playground to explore pairs.	 Explore filling egg boxes in pair wise and five wise using practical resources begin to link this to visual representations using a tens frame.	
Find a double - understand double as twice as many	 Recognise what is meant by the term double 	 Represent doubles on their fingers	

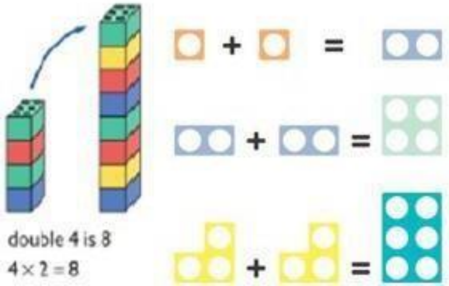
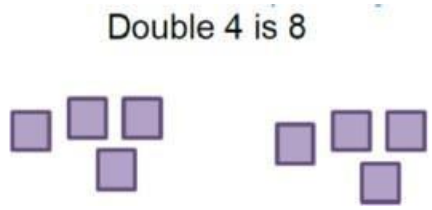
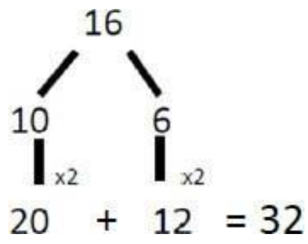
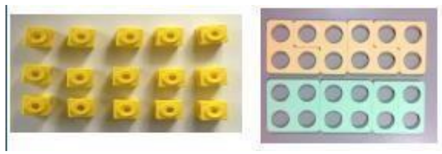
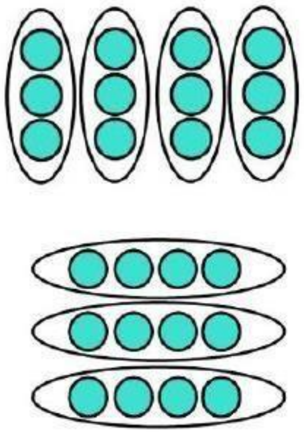
	Explore doubles using numicon and other mathematical equipment.   Play games involving doubling e.g. using dice or caterpillar doubles Play doubling barrier games	 Use a tens frame to represent a double	

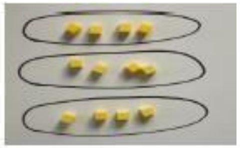
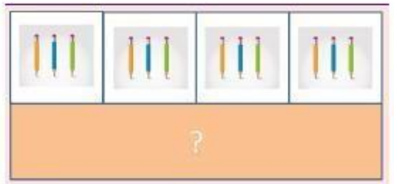
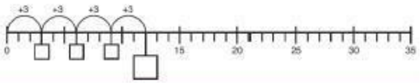
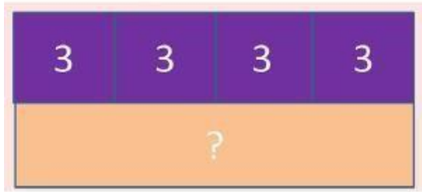
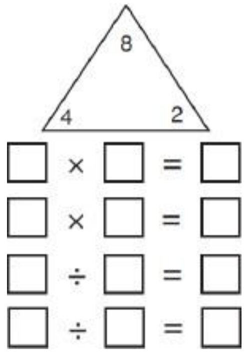
Multiplication - Year 1

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling numbers.	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 
Counting in multiples.	Count the group as children are skip counting, children may use their fingers to help. 	Children make representations to show counting in multiples.  	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25, 30

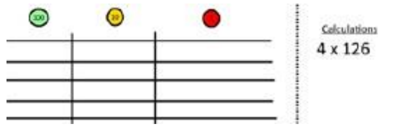
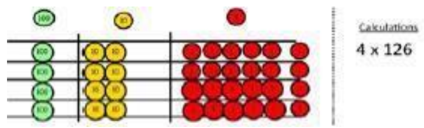
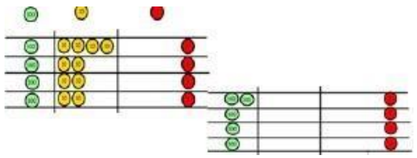
Repeated grouping/repeated addition.	**3** groups 4 = **4** + 4 + 4 = There are 3 equal groups, with 4 in each group. 	Children to represent the practical resources in a picture and use a bar model. 	3 groups 4 = 12 4 + 4 + 4 = 12
Understanding arrays.	Use objects laid out in arrays to find the answers to 2 lots of 5, 3 lots of 2s. 	Draw representations of arrays to demonstrate understanding. 	3 groups 2 = 6 2 groups 5 = 10

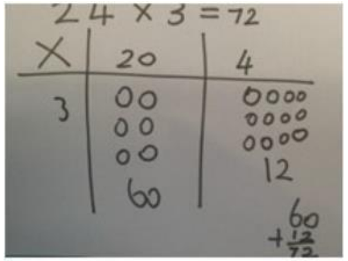
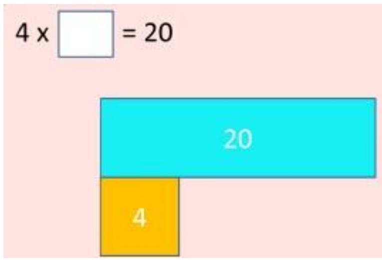
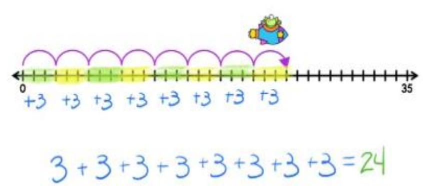
Multiplication - Year 2

Objective & Strategy	Concrete	Pictorial	Abstract
Doubling numbers.	Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling. 	Draw pictures to show how to double numbers. 	Partition a number and then double each part before recombining it back together. 
Multiplication is commutative.	Create arrays using counters, cubes  and numicon.  Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not change the answer.	Use representations of arrays to show different calculations and explore commutativity. 	$12 = 3 \times 4$ $12 = 4 \times 3$ Use an array to write multiplication sentences and reinforce repeated addition.  $5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$

	 		
Counting in multiples of 2, 5 and 10 from 0. (repeated addition)	Count the groups as children are skip counting, children may use their fingers to help. Progress onto bar models.  5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40   	Number lines, counting sticks and bar models should be used to show representation of counting in multiples.     	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8, 10 0, 3, 6, 9, 12, 15 0, 5, 10, 15, 20, 25, 30 4 x 3 =
Using the inverse. (this should be taught alongside division, so pupils learn how the two operations work alongside each other)		 	2 x 4 = 8 4 x 2 = 8 8 ÷ 2 = 4 8 ÷ 4 = 2 8 = 2 x 4 8 = 4 x 2 2 = 8 ÷ 4 4 = 8 ÷ 2 Show all 8 related fact family sentences.
Multiplication knowledge	Have a rapid recall of the: • 2x table • 5x table • 10x table		

Multiplication Year 3

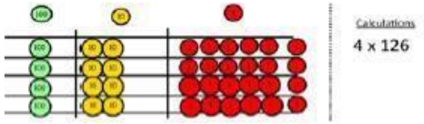
<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>The grid method.</td>
<td>Show the links with arrays to first introduce the grid method.
Move onto place value counters to show how we are finding groups of a number. We are multiplying by 4, so we need 4 rows…

Fill each row with 126…

Add up each column, starting with the ones making any exchanges needed.
</td>
<td>Children can represent their work with place value counters in a way that they understand.

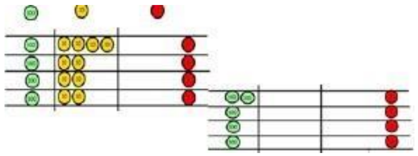
They can draw the counters using colour to show different amounts or just use the circles with 10s or 1s in the different columns to show their thinking.

Bar models are used to explore missing numbers.

Number lines used to develop understanding of multiplication as repeated addition
</td>
<td>Begin with multiplying by one digit numbers and showing the clear addition alongside.
<table><tr><td>×</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table>
210 + 35 = 245</td>
</tr>
</table>

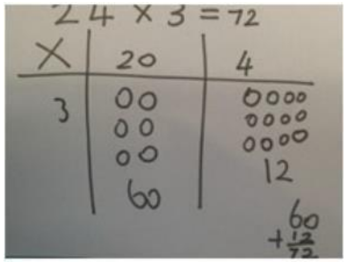
Rapid Recall (multiplication and division	Multiplication and division facts for the: • 2 x tables • 5 x table • 10, x table • 3 x table • 4 x table • 8 times tables. *<u>Strategies</u>* • *'Double -double'/'half -half' links within the listed times tables.* • *Associativity (pushing numbers around)* • *Using what I already know*

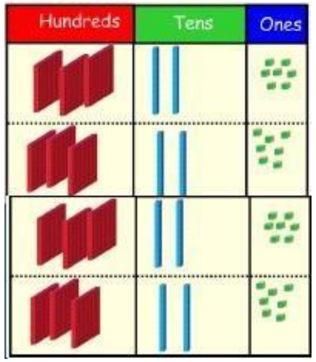
Multiplication Year 4

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>The grid method (recap from Year 3 for 2 digit x 1 digit).

Children progress to multiplying 3 digit numbers by 1 digit (Year 4 expectation).</td>
<td>Use place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows.


Fill each row with 126.

Add up each column, starting with the ones making any exchanges needed.</td>
<td>Children can represent their work with place value counters in a way that they understand.

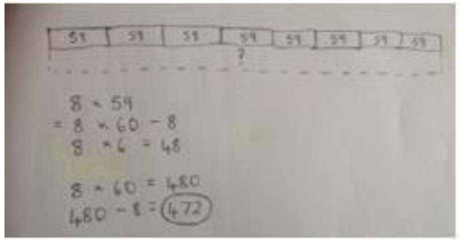
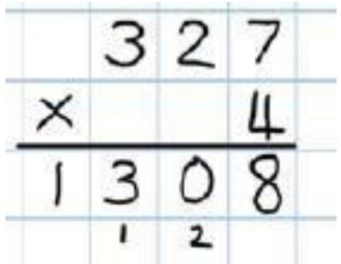
They can draw the counters using colour to show different amounts or just use the circles in the different columns to show their thinking.

</td>
<td>Multiply 3 digit by 1 digit numbers using the grid method.
<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>1200 + 80 + 28 = 1,308</td>
</tr>
</table>

<table>
<tr>
<td>Column Multiplication.</td>
<td>Children can continue to be supported by place value counters at this stage of


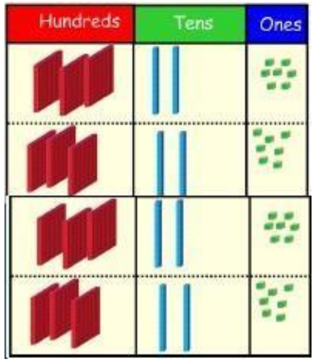
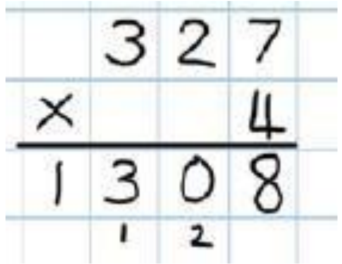
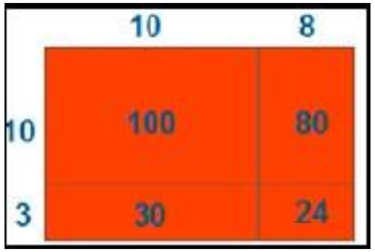
multiplication. This is initially done where there is no regrouping.

321 x 2 = 642

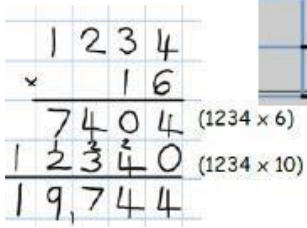
It is important at this stage that they always multiply the ones column first.

The corresponding long multiplication is modelled alongside this method.</td>
<td>The grid method may be used to show how this relates to a formal written method (see abstract column).
<table>
<tr><td>x</td><td>300</td><td>20</td><td>7</td></tr>
<tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr>
</table>
Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.
</td>
<td>The grid method can then be progressed onto the compact method.

</td>
</tr>
<tr>
<td>Rapid Recall (multiplication and division</td>
<td colspan="3">Multiplication and division facts for all of the times tables up to 12 x 12 and related division facts.</td>
</tr>
</table>

Multiplication Year 5

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Multiplication (3 and 4 digits x 1 digit).</td>
<td>Children can continue to be supported by place value counters at this stage of multiplication. This is initially done where there is no regrouping.

</td>
<td>The grid method may be used to show how this relates to a formal written method (see abstract column).
<table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table></td>
<td>The grid method can then be progressed onto the compact method.

</td>
</tr>
<tr>
<td>Column Multiplication - Long multiplication.</td>
<td></td>
<td>

Continue to use bar modelling to support problem solving.
Make look like Year 3 version</td>
<td>Expanded method to be added in
for long multiplication.

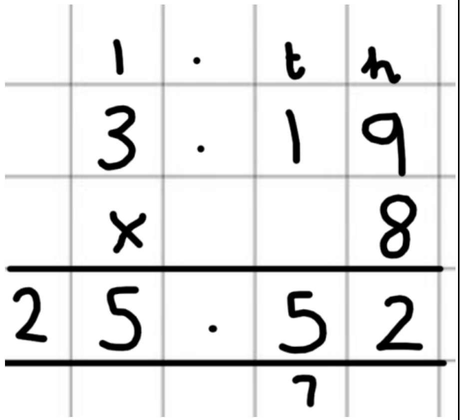



Progress to using the column method.</td>
</tr>
</table>

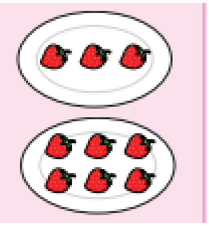
Rapid Recall (multiplication and division	Square numbers to 144. Cube numbers. Establish whether a number is prime Recall all prime numbers up to 19 <u>*Strategies*</u> - *X by9* - *X/÷ by 10/100/1000 – including decimals* - *Use what you knowto…* - *x/÷ by5/50/25* - *x by ½* - *Use factor pairs – 24 x 16*

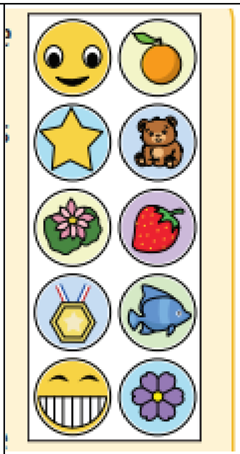
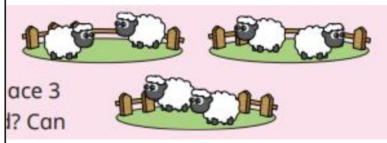
Multiplication Year 6

<table>
<tr><th>Objective & Strategy</th><th>Concrete</th><th>Pictorial</th><th>Abstract</th></tr>
<tr>
<td>Column Multiplication
Long multiplication.</td>
<td>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</td>
<td><table><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table>
Continue to use bar modelling to support problem solving.</td>
<td>Progress to using the column method for long multiplication.

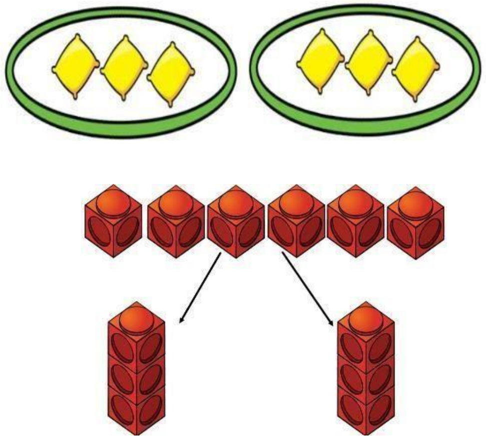
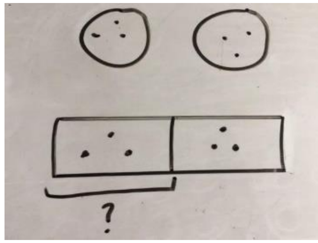
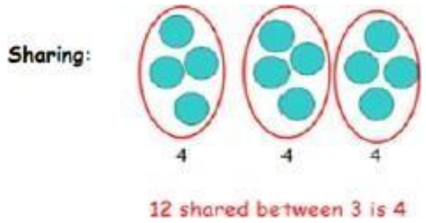
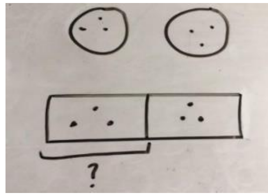

</td>
</tr>
<tr>
<td>Multiplying decimals up to 2 decimal places by a single digit.</td>
<td></td>
<td></td>
<td>Remind children that the single digit belongs in the ones column. Line up the decimal points in the question and answer.
</td>
</tr>
</table>

Division - Year R

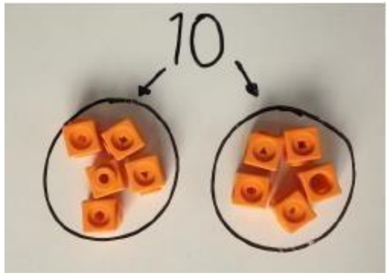
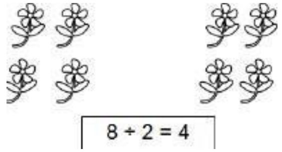
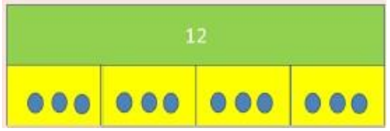
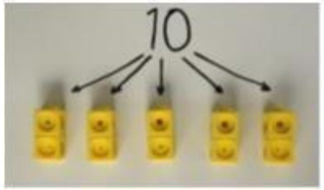
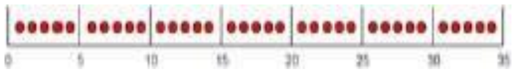
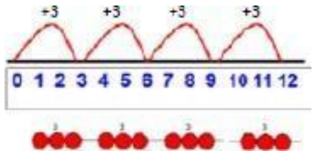
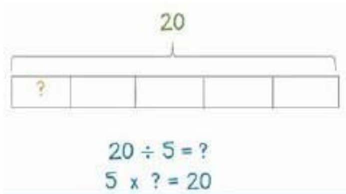
Objective & Strategy	Concrete	Pictorial	Abstract
Introducing sharing equally	 Using practical resources to share items equally e.g. Goldilocks and the Three Bears. Modelling sharing equally and unequally - is this fair? Use of puppets to model  Modelling sharing playdough - is this shared out equally? Preparing for a game - is it fair if ...has all the swords for pop up pirate? Read stories such as: the Squirrels who Squabbled and discuss fairness. Begin to understand the idea of odd and even sharing		

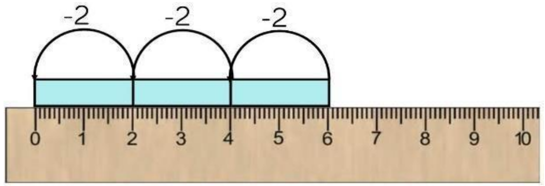
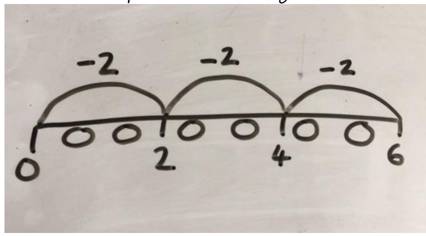
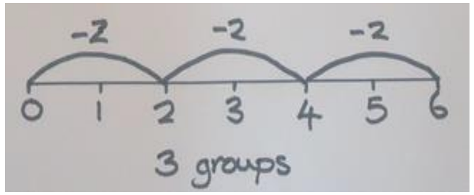
Exploring grouping	 Explore a story such as: Ness the Nurse and enact a familiar scenario with the children. I have 10 stickers. Give 1 sticker each to 7 children and 3 to 1 child. Is this grouped equally?  Ask children to explore grouping small world characters e.g. 6 sheep into 3 fields Discussions in PE I have 12 children - how many groups of 3 can I make? How many groups of 4. Using playdough to explore adding 3 buttons to each Gingerbread man.		
Find a half- understand that halving is two equal groups that make the whole	Children learn to share class resources equally with a friend during play. Children share food items during role play, such as the pizza and cake, between two.		

Division Year 1

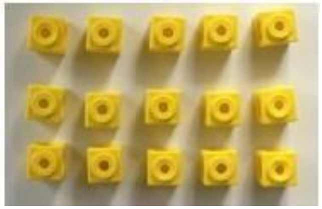
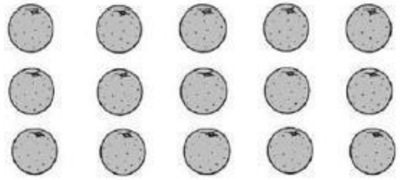
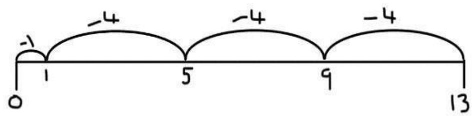
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing Halving- equal and unequal groups	Sharing using a range of objects: Share 6 into 2 groups 	Use pictures or shapes to share quantities:  	Children continue with pictorial method until fully secure. Children should also be encouraged to use their 2 times tables facts.  To progress further, children can then be moved onto: '6 shared between 2 is 3'

Division Year 2

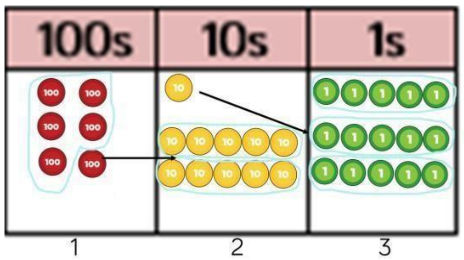
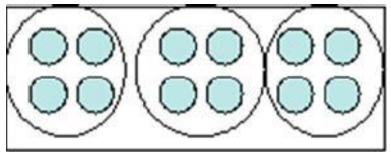
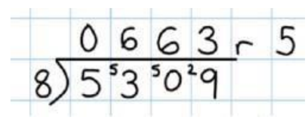
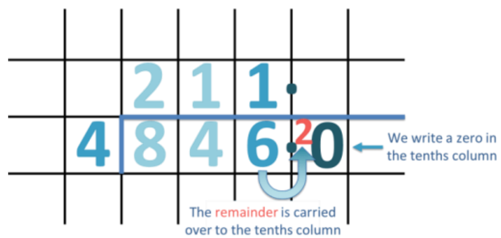
Objective & Strategy	Concrete	Pictorial	Abstract
Division as sharing	I have 10 cubes, can you share them into 2 equal groups?  	Children use pictures or shapes to share quantities:  Children use bar modelling to show and support understanding: $12 \div 4 = 3$  	$12 \div 3 = 4$
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.  	Use number lines for grouping:  Use bar model to support with division:  	$28 \div 7 = 4$ Divide 28 into 7 groups. How many are in each group?

Division using repeated subtraction	Repeated subtraction using Cuisenaire rods above a ruler: $6 \div 2$  3 groups of 2	Children to represent repeated subtraction pictorially:  	Abstract number line to represent the equal groups that have been subtracted: 

Division Year 3

Objective & Strategy	Concrete	Pictorial	Abstract
Division with arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created:  15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences:  15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Find the inverse of multiplication and division sentences by creating eight linking number sentences: 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7 28 = 7 x 4 28 = 4 x 7 4 = 28 ÷ 7 7 = 28 ÷ 4
Division with remainders			13 ÷ 4 = 3 remainder 1 Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line:  '3 groups of 4, with 1 left over'

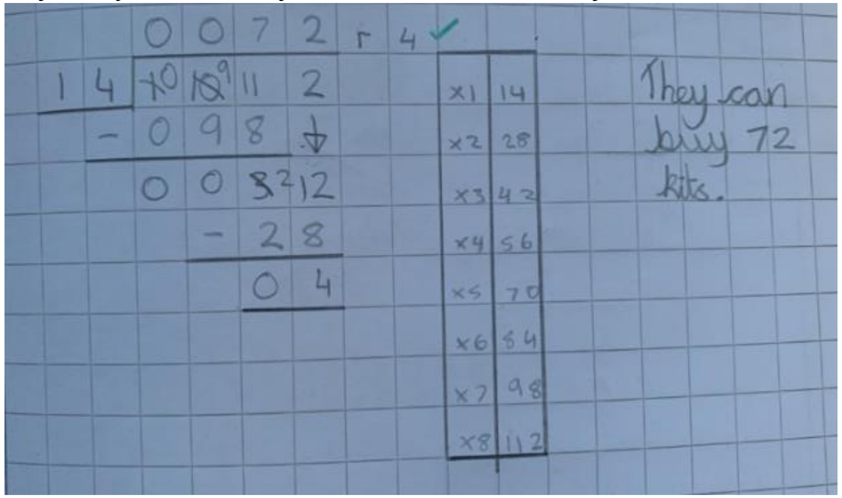
Year 4 - 6 Division

Objective & Strategy	Concrete	Pictorial	Abstract
Short division with a remainder <u>Year 4</u> Up to 3 digits by 1 digit <u>Year 5</u> Up to 4 digits by a 1 digit with remainders <u>Year 6</u> Up to 4 digits by a 1 digit and then progress to long division (next objective)	Short division using place value counters to group: 615 ÷ 5  1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones?	Children can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups:  However, children should be encouraged to move towards counting in multiples to divide more efficiently.	Begin with divisions that divide equally with no remainders: 2 1 8 4 | 8 7 ^{3}2 Move onto divisions with a remainder: 8 6 r 2 5 | 4 3 ^{3}2  Year 5/6 Children can then progress onto expressing the remainder as fractions (e.g. 5/8) and decimals (e.g. 663.625). 846 ÷ 4 

Long division with remainder

Year 6 Division

Begin by modelling method with a 1-digit divisor.



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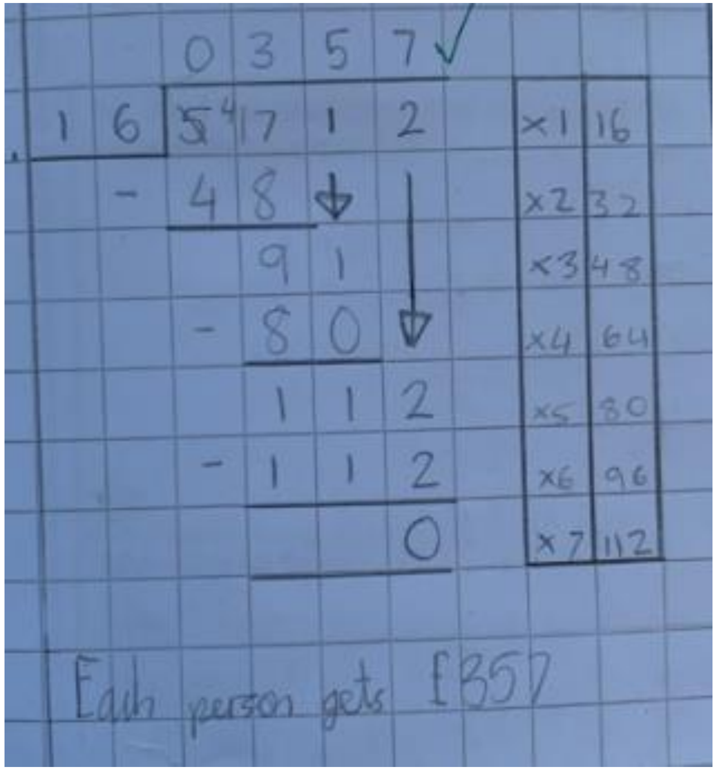


Divide - the number inside the house with the number outside of the house. Put the answer on top.

Multiply -the number outside of the house by the number on top of the house. Put this answer below the number inside the house.

Subtract - the number inside the house from the number below the inside number.

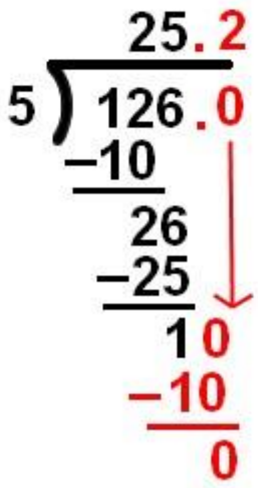
Bring down - the next number in the dividend.

Repeat - all the steps repeated as many times as needed until you get down to 0.
If there is a leftover this is your remainder.

When moving onto using long division with a 2-digit divisor, children can write out multiples first:

2 2)6 7 1 0

22, 44, 66, 88, 110
etc

Long division with decimal remainders	 When there is a remainder which you need to write as a decimal, bring down the 0 in the from then tenths column, and repeat the process as before.	